SYLLABUS FOR M.SC.
IN
MATHEMATICS

Applicable for the Session 2016-2017 onwards

Department of Mathematics
West Bengal State University
Berunanpukuria, P.O. - Malikapur
Barasat, Kolkata-700 126
West Bengal, India

WEST BENGAL STATE UNIVERSITY

M.SC. MATHEMATICS COURSE STRUCTURE & MARK DISTRIBUTION WITH CREDIT POINTS

Semester	Paper Code	Title of the Paper	Lectures in (hrs.) per week		Marks			Credit Points
			L	P	(CA)	(ETE)	Total	
I	MAT 215111	Algebra *	4		10	40	50	4 CP
	MAT 215112	Linear Algebra *	4		10	40	50	4 CP
	MAT 215113	Real Analysis *	4		10	40	50	4 CP
	MAT 215114	Complex Analysis *	4		10	40	50	4 CP
	MAT 215115	Mechanics *	4		10	40	50	4 CP
	MAT 215117 (Practical)	Computer Programming in C (Practical) *		4	5	20	25	2 CP
II	MAT 215121	Topology *	4		10	40	50	4 CP
	MAT 215122	Measure Theory *	4		10	40	50	4 CP
	MAT 215123	Functional Analysis *	4		10	40	50	4 CP
	MAT 215124	Differential Eq. & Generalized functions *	4		10	40	50	4 CP
	MAT 215125	Operations Research & Numerical Analysis *	4		10	40	50	4 CP
	MAT 215127 (Practical)	Numerical Analysis with Computer Application (Practical) *		4	5	20	25	2 CP
III	MAT 215231	PDE & Integral Transform *	4		10	40	50	4 CP
	MAT 215232	Differential Geometry *	4		10	40	50	4 CP
	MAT 215233	Theory of computations & Graph Theory *	4		10	40	50	4 CP
	MAT 215234	Minor Elective I **	4		10	40	50	4 CP
	MAT 215236 (Practical)	Mathematical Modeling (Practical)		4	5	20	25	2 CP
IV	MAT 215241	Major Elective I ****	4		10	40	50	4 CP
	MAT 215242	Major Elective II ****	4		10	40	50	4 CP
	MAT 215243	Minor Elective II **	4		10	40	50	4 CP
	MAT 215245	Project / Dissertation					50	4 CP
	MAT 215246	Seminar Assignment / Comprehensive Viva					25	2 CP
		GRAND TOTAL					1000	80 CP

L : Lecture; P : Practical; CA : Continuous Assessment; ETE : End Term Examination; CP : Credit Points ; Papers marked * are Core Courses ; Papers marked **, *** & ****are Elective (Minor , Major I & Major II respectively) Courses

Minor & Major Elective Papers : Some of the following topics are being offered for Minor Elective I ** & Minor Elective II **, Major Elective I *** & Major Elective II **** Courses. New Topics may be added to the list from time to time on availability of subject experts.

Minor Elective Course I **

Any One Subject to be chosen either from **Group A** or from **Group B.**

Group A

1. Operator Theory and Banach Algebra
2. Number Theory
3. Lie Groups & Lie Algebras

Group B.

1. Differential Equations and Dynamical System
2. Continuum Mechanics
3. Integral Equations and Boundary Value Problems

Minor Elective Course II **

Any one subject out of the following to be chosen:

1. Algebraic Topology and Category Theory
2. Fluid Dynamics

Major Elective Course I*** & Major Elective Course II****

Each year, Department will offer some modules from the following list of modules, subject to the availability of resources. Two topics (one for Major Elective Course I*** & another for Major Elective Course II****) have to be chosen by a candidate from the offered modules, keeping in view the prerequisites and suitability of the combination.

1. **Advanced Topology I**
2. **Advanced Topology II**
3. **Advanced Complex Analysis**
4. **Advanced Functional Analysis**
5. **Advanced Real analysis**
6. **Harmonic Analysis / or Abstract Harmonic Analysis**
7. **Representation Theory of Groups**
8. **Fuzzy Sets and Their Applications**
9. **Differential Topology**
10. **Fundamentals of Computer Science Theory and Practicals**
11. **Algebraic Geometry**
12. **Commutative Algebra**
13. **Non- Commutative Rings**
14. **Non-Linear Differential Equations And Dynamical System**
15. **Non-Linear Dynamics & Chaos**
16. **Fluid Plasma Theory**
17. **Kinetic Plasma Theory**
18. **Magnetohydrodynamics**
19. **Quantum Mechanics**
20. **Mathematics of Finance & Insurance**
21. **Fundamentals of Mathematical Biology**

Note : Other Topics according to the availability of subject experts may be added.

Detailed M.Sc. Syllabus
WEST BENGAL STATE UNIVERSITY
Department of Mathematics

Semester	Paper Code	Title of the Paper	Total Marks	Credit Points
I	MAT 215111	Algebra *	50	4 CP
	MAT 215112	Linear Algebra *	50	4 CP
	MAT 215113	Real Analysis *	50	4 CP
	MAT 215114	Complex Analysis *	50	4 CP
	MAT 215115	Mechanics *	50	4 CP
	MAT 215117 (Practical)	Computer Programming in C (Practical) *	25	2 CP
II	MAT 215121	Topology *	50	4 CP
	MAT 215122	Measure Theory *	50	4 CP
	MAT 215123	Functional Analysis *	50	4 CP
	MAT 215124	Differential Eq. & Generalized functions *	50	4 CP
	MAT 215125	Operations Research & Numerical Analysis *	50	4 CP
	MAT 215127 (Practical)	Numerical Analysis with Computer Application (Practical)*	25	2 CP
III	MAT 215231	PDE & Integral Transform *	50	4 CP
	MAT 215232	Differential Geometry *	50	4 CP
	MAT 215233	Theory of computations & Graph Theory *	50	4 CP
	MAT 215234	Minor Elective I **	50	4 CP
	MAT 215236 (Practical)	Mathematical Modeling (Practical)	25	2 CP
IV	MAT 215241	Major Elective I***	50	4 CP
	MAT 215242	Major Elective II ****	50	4 CP
	MAT 215243	Minor Elective II **	50	4 CP
	MAT 215245	Project / Dissertation	50	4 CP
	MAT 215246	Seminar Assignment / Comprehensive Viva	25	2 CP
		GRAND TOTAL	1000	80 CP

PROGRAM SPECIFIC OUTCOMES

Successful completion of the two-year M.SC course in Mathematics will enable the students to

1. Approach and analyse the problems arising in their chosen careers in a logical manner and apply these skills to any real-life situation.
2. Apply computational and modelling skills to specific tasks , especially in the emerging and developing processes and industries.
3. Independently pursue research work in any area of Pure or Applied Mathematics ; work in a group confidently and contribute significantly to any research project.
4. Acquire a systematic knowledge of fundamental aspects of various branches of Mathematics which would help them in qualifying National and State-level examinations
5. Think and analyse independently, and apply their skills in mathematical logic to any profession of their choice.
6. Take up pedagogy in Mathematics or related subjects if they are so inclined.

Semester : I

Course : 215111

Algebra : 50 Marks (4 CP) (Duration : 6 Months)

Syllabus :

1. **Group Theory**
   Group action, permutation representation and Caley's theorem. Conjugacy classes and class equation, p-groups. Converse of Lagrange's theorem for finite abelian groups. Sylow's theorems and its applications. Direct product, finitely generated abelian groups.
   Solvable groups – solvability of S_n .
2. **Ring Theory**
   Subrings and ideals, principal ideals, Principal Ideal Domain (PID). Quotient ring, isomorphism and correspondence theorems. Prime, primary and maximal ideals – examples, characterizations and their interrelations. Divisor, common divisor and greatest common divisor; prime and irreducible. elements, characterizations of prime and maximal ideals in terms of prime and irreducible elements. Factorization Domain (FD) and Unique Factorization Domain (UFD).
   Ring with chain conditions – Noetherian rings and Artinian rings (definition only).
   Polynomial ring.
3. **Field Extension and Galois Theory**
   Field extension – algebraic and transcendental extension and their characterizations.
   Splitting field and algebraic closure. Separable and normal extension.
   Cyclotomic polynomial and Galois field. Galois theory – introduction, basic ideas and results focusing the fundamental theorem of Galois theory.
   Solvability by radicals.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following:

i) Sylow's theorems and its applications, Solvable groups,
ii) Prime, primary and maximal ideals,
iii) Jacobsons radical, semisimple ring, Unique Factorization Domain,
iv) Basics of Field extension & Galois theory.

Also there is a scope, for applying the acquired knowledge of the above algebraic methods/ tools, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References :

1. D.S. Malik, John M. Mordeson and M.K. Sen ,Fundamentals of Abstract Algebra, McGraw Hill, International Edition, 1997.
2. Dummit and Foote , Abstract Algebra, John Wiley and Sons, Inc.
3. T. H. Hungerford, Algebra , Springer Verlag
4. John B. Fraleigh , A first course in Abstract Algebra , Narosa.
5. I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd, New Delhi, 1975
6. S. Lang, Abstract Algebra , 2^{nd} edition, Addition -Wesley .

Course : 215112

Linear Algebra : 50 Marks (4 CP) (Duration : 6 Months)

Syllabus :

Modules, Submodules, Quotient Modules, Morphisims, Isomorphisim Theorems, Correspondence Theorem, Simple Modules, Free modules, Noetherian and Artinian Modules, Exact Sequence,Dual Modules, Fundamental Structure Theorem for Fintely Generated Modules over PID- Statement only.

Matrices and Linear Transformations, Representation of Linear Transformations by Matrices,Rank—Nulity Theorem, Linear Functionals, Dual Spaces,Dual Basis.

Matric Polynomial, Minimal Polynomial, Characteristic Polynomials & Characteristic Roots. Diagonalization Of Matrices, Reduction to Triangular Forms, Jordan Blocks,

Jordan Canonical Forms , Invariant Factors, Rational Canonical Forms, Smith Normal Form Over a PID.

Bilinear Forms , Quadratic Forms, Hermitian Forms, Positve Definite Forms & Matrices, Pricipal Minor Criteron, Direct Sum Decomposition, Signature, Sylvester Law Of Inertia, Simultaneous Reduction of Pair of Forms .

Course Outcomes: On completion of this course , the students will be able to identify , analyze, classify, demonstrate and explain the acquired knowledge on the following :

i) Modules with chain conditions(Noetherian and Artinian), Dual Modules, Free Modules,
ii) Dual Spaces, Dual Basis, Dimension of Quotient space,
iii) Minimal Polynomial, Diagonalization of Matrices, Reduction to Triangular Forms,
iv) Jordan Canonical Forms, Rational Canonical Forms, Smith Normal Form,
v) Bilinear Forms , Quadratic Forms, Hermitian Forms,
vi) Direct sum decomposition theorem, Pricipal Minor Criteron,
vii) Sylvester Law Of Inertia, Simultaneous Reduction of Pair of Forms.

Also there is a scope, for applying the acquired knowledge of the above linear algebraic methods/ tools, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References :

1. M. Artin, Algebra, Prentice Hall of India, 1994.
2. K. Hoffman and R. Kunze, Linear Algebra, Pearson Education (India), 2003. Prentice-Hall of India, 1991.
3. S. Lang, Linear Algebra, Undergraduate Texts in Mathematics, Springer-Verlag, New York, 1989.
4. A.R. Rao, P. Bhimashankaram, Linear Algebra. (Tata Mc-Graw Hill)
5. P. Lax, Linear Algebra, John Wiley & Sons, New York,. Indian Ed. 1997
6. H.E. Rose, Linear Algebra, Birkhauser, 2002.
7. S. Lang, Algebra, 3rd Ed., Springer (India), 2004.
8. G. Strang : Linear Algebra & its Applications, Harcourt Brace Jovanichn 3rd Edition 1998.
9. B. Noble and J.W. Daniel. Applied Linear Algebra, third edition, 1988.Prentice Hall, NJ.
10. N.J. Pullman. Matrix Theory and its Applications, 1976. Marcel Dekker Inc. New York.
11. I. N. Herstein, Topics in Algebra.
12. R. Stall, Linear Algebra and Matrix Theory
13. Evar D. Nering, Linear Algebra and Matrix Theory
14. B. C. Chatterjee, Linear Algebra
15. Rudra Pratap : Gettiing started with MATLAB 7, Oxford Press, Indian edition, 2007.

Course : 215113

Real Analysis : 50 Marks (4 CP) (Duration : 6 Months)

Syllabus :

The Lebesgue measure **:** Definition of the Lebesgue outer measure and inner measure. Measurable functions **:** Definition on a measurable set in R and basic properties, Simple Functions.

Functions of bounded variation **:** Definition and basic properties, Lipschitz condition, Jordan decomposition, Nature of points of discontinuity, Nature of points of non-differentiability.

Absolutely continuous functions **:** Definition and basic properties, Deduction of the class ofall absolutely continuous functions as a proper subclass of all functions of bounded variation; Characterization of an absolutely continuous function in terms of its derivative vanishing almost everywhere.

Riemann-Stieltjes integral **:** Existence and basic properties, Integration by parts, Integration of a continuous function with respect to a step function, Convergence theorems in respect of integrand.

Differentiation on R^n **:** Directional derivatives and continuity, the total derivativeand continuity, total derivative in terms of partial derivatives, the matrix transformation of $T : R^n \rightarrow R^n$. The Jacobian matrix.

The chain rule and its matrix form. Mean value theorem for vector valued function. Mean value inequality. A sufficient condition for differentiability. A sufficient condition for mixed partial derivatives. Functions with non-zero Jacobian determinant, the inverse function theorem, the implicit function theorem as an application of Inverse function theorem.

Extremum problems with side conditions – Lagrange's necessary conditions as an application of Inverse function theorem.

Integration on R^n **:** Integral of $f : A \rightarrow R$ when $A \subset R^n$ is a closed rectangle. Conditions of integrability.Integrals of $f : C \rightarrow R$, $C \subset R^n$ is not a rectangle, concept of Jordan measurability of a set in R^n.Fubini's theorem for integral of $f : A \times B \rightarrow R$, $A \subset R^n$, $B \subset R^n$, are closed rectangles. Fubini's theorem for $f : C \rightarrow R^n$, $C \subset A \times B$. Formula for change of variables in an integral in R^n.

Course Outcomes: Upon completion of this course, to understand the basics of Real Analysis and improve the logical thinking.

References :

1. T. M. Apostol : Mathematical Analysis.
2. M. Spivak : Calculus on Manifolds.

Course : 215114

COMPLEX ANALYSIS : 50 Marks (4 CP) (Duration : 6 Months)

Syllabus :

Riemann's sphere, point at infinity and the extended complex plane.

Functions of a complex variable, limit and continuity. Analytic functions, Cauchy-Riemann equations. Complex integration, Cauchy's fundamental theorem (statement only) and its consequences. Cauchy's integral formula. Derivative of an analytic function, Morera's theorem, Cauchy's inequality, Liouville's theorem. Fundamental theorem of classical algebra.

Uniformly convergent series of analytic functions. Power series. Taylor's theorem. Laurent's theorem.

Zeros of an analytic function. Singularities and their classification. Limit points of zeros and poles. Riemann's theorem. Weierstrass-Casorati theorem. Theory of residues. Argument principle. Rouche's theorem. Maximum modulus theorem. Schwarz lemma. Behaviour of a function at the point at infinity.

Contour integration. Conformal mapping, Bilinear transformation. Idea of analytic continuation. Multivalued functions – branch point. Idea of winding number.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following:

i) Stereographic Projection, Riemann's sphere, point at infinity, extended complex plane,

ii) Cauchy-Goursat Theorem, Cauchy's integral formulas, Morera's theorem, Liouville's theorem,
iii) Fundamental theorem of classical algebra, Schwarz Reflection Principle, Maximum Modulus Principle,
iv) Cauchy-Hadamard Theorem, Taylor's theorem and Laurent's theorem,
v) Riemann's Removal singularity thorem, Weierstrass-Casorati,
vi) The Cauch's Residue Theorem, Argument principle and their applications,
vii) Conformal mapping, Bilinear transformation, Idea of analytic continuation.

Also there is a scope, for applying the acquired knowledge of the above concepts/ methods/ tools in Complex Analysis, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References :

1. A. I. Markushevich **:** Theory of Functions of a Complex Variable(Vol. I, II and III).

2. R. V. Churchill and J. W. Brown **:** Complex Variables and Applications.

3. E. C. Titchmarsh **:** The Theory of Functions.

4. E. T. Copson **:** An Introduction to the Theory of Functions of a Complex Variable.

5. J. B. Conway **:** Functions of One Complex Variable.

6. L. V. Ahlfors **:** Complex Analysis.

7. H. S. Kasana **:** Complex Variables – Theory and Applications.

8. Shanti Narayan and P. K. Mittal **:** Theory of Functions of a Complex Variable.

9. A. K. Mukhopadyay **:** Functions of Complex Variables and Conformal Transformation.

10. J. M. Howie **:** Complex Analysis.

Course : 215115

Mechanics : 50 Marks (4 CP) (Duration : 6 Months)

Syllabus :

Newton'Laws of Motion. Single and many particle systems. Concepts of energy, momentum, angular momentum, conservation laws.

The Lagrangian formulation : Constraints, generalized coordinates, holonomic and nonholonomic constraints, Lagranges equations. The Hamiltonian, Noether's theorem, Hamiltons Equations.

Small oscillations and stability.

Motion of a rigid bodies, Euler's theorem. Inertia tensor, Euler angles, Euler's equations. Torque free motion. Motion of a free top, Motion of a heavy symmetrical top.

Hamilton's formulation, Hamilton's principle, Hamilton's equations, Legendre transformation, some conservation laws, Liouville's theorem, Poisson brackets, canonical transformation, generating function, action-angle variables, Hamilton Jacobi equation.

The Special Theory of Relativity, Galilean transform, length contraction, time dilation, Lorentz transform and consequences.

Course Outcomes: Students will be able to apply the equations of motion to solve analytically the problems of motion of a single particle/a system of particle or rigid body under conservative force fields.

2. Use the Hamilton's principle for deriving the equations of motion of a system.
3. Gain knowledge of Hamiltonian system and phase planes from the point of view of mechanics.
4. Use the theory of normal modes for solving problems related to oscillations and vibrations.
5. Students will learn the basics of classical mechanics and STR required for further studies in solid and quantum mechanics.

References :

1.Classical Mechanics. Herbert Goldstein.
2.Treatise of the analytical Dynamics of particles and rigid bodies. E T Whittaker.
3. Classical Mechanics Rana and Joag.
4. Analytical Mechanics, Fowles and Cassiday.

<u>Course: 215 117</u>

Computer Programming in C (Practical): 25 Marks; 2 CP (Duration : 6 Months)

Syllabus :

C character set, data types, operators and expressions, input and output operators, control statements,functions, arrays – single and multi-dimensional, strings, pointers, structures and unions, data files. Scope of the variable.

Stacks, queues and linked lists. Sorting and searching algorithms.

Course Outcomes: At the end of this course a student should be able to :

- understand the purpose of basic computer programming language,
- understand and apply control statements, implementation of arrays, functions, etc.,
- enhance program writing skills for solving several real life and Mathematical problems.

References :

1. Programming with C – Byron S Gottfried

2. The C Programming Language – Brian W Kernighan , Dennis M Ritchie

3. Programming in ANSI C – E Balagurusamy

Semester : II

Course: 215 121

Topology : 50 Marks (4 credit points) (Duration : 6 Months)

Syllabus :

Brief Description of

Countable and uncountable sets. Axiom of choice and its equivalence. Cardinal and ordinal numbers. Schroeder-Bernstein theorem. Continuum hypothesis. Zorn's lemma and well-ordering theorem. Ordinal Numbers. The first uncountable ordinal.

Fundamentals of Topological spaces

Definition and examples; open and closed sets. Bases and sub-bases. Closure and interior – their properties and relations; exterior, boundary, accumulation points, derived sets, dense set, G_δ and F_σ sets. Neighboourhoods and neighbourhood system. Subspace and induced / relative topology. Relation of closure, interior, accumulation points etc. between the whole space and the subspace.

Alternative methods of defining a topology in terms of Kuratowski closure operator, interior operator, neighbourhood systems.

Continuous, open and closed maps, pasting lemma, homeomorphism and topological properties.

Countability axioms

1^{st} and 2^{nd} countability axioms, Separability,Lindeloffness. Characterizations of accumulation points, closed sets, open sets in a 1^{st} countable space w.r.t. sequences .Heinei's continuity criterion.

Separation Axioms

T_i spaces(i = 0, 1, 2, 3, 3½, 4, 5), their characterizations and basic properties.

Urysohn's lemma and Tieze' extension theorem (statement only) and their applications.

Connectedness

Connected and disconnected spaces. Connectedness on the real line. Components and quasi-components. Local connectedness.

Compactness

Compactness and some of its basic properties. Compactness and FIP. Continuous functions and compact sets. Equivalence of compactness, countable compactness and sequential compactness in metric spaces.

Product spaces

Product and box topology, Projection maps. Alexander subbase theorem and Tychonoff product theorem. Separation and product spaces. Connectedness and product spaces.Countability and product spaces.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge on the following :

i) Axiom of choice, Continuum hypothesis, Cardinal and Ordinal numbers,
ii) Basics of Topological spaces, Relative topology, homeomorphism and topological properties,
iii) Alternative methods of defining a topology in terms of Kuratowski closure operator, interior operator and neighbourhood systems,
iv) Countability axioms, Heinei's continuity criterion,
v) Lower & higher separation axioms, Urysohn's lemma and Tieze's extension theorem (statement only) and their applications,
vi) Connected and disconnected spaces, path connected spaces, Compactness, Alexander subbase theorem, equivalence of various compactness in metric spaces,
vii) Product and box topology, Tychonoff product theorem.

Also there is a scope, for applying the acquired knowledge of the above topological methods/ tools, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References :

1. General Topology by J. L. Kelley, Van Nostrand
2. General Topology by S. Willard, Addison-Wesley
3. Topology by J. Dugundji, Allyn and Bacon
4. Topology, A first course by J. Munkres, Prentice Hall, India
5. Introduction to topology and modern analysis by G. F. Simmons, McGraw Hill
6. Introduction to General topology by K. D. Joshi, Wiley Eastern Ltd.
7. General Topology by Engelking, Polish Scientific Publishers, Warszawa
8. Counter examples in Topology by L. Steen and J. Seebach
9. A text book of Topology by B. C. Chatterjee, S. Ganguly and M. Adhikari, Asian Books Pvt.

Course: 215 122

Measure Theory : 50 Marks (4 CP) (Duration : 6 Months)

Syllabus :
Outer Lebesgue Measure in E^* (starting with the concept of length of an interval); the properties of outer Lebesgue Measure m^*.

Outer measure $\boldsymbol{\mu}^*$ on S-where S is a space; the concept of μ-measurable sets with the help of $\boldsymbol{\mu}^*$. Necessary and sufficient condition for μ-measurability. Properties of **μ**-measurable sets. The structure of μ-measurable sets-the concept of σ-algebra; the σ-algebra of Lebesgue measurable sets.

Properties of Lebesgue measure , Vitali's theorem: The existence of an non-measurable set. The Borel sets & Lebesgue measurable sets- a comparison

μ-measurable functions, their properties, characteristic functions; step functions.

Lebesgue Integration

The following theorems: i) Let f be a nonnegative measurable function. For each set M ЄM (M is the **σ**- algebra of μ-measurable sets) define $\nu_f(M) = \int_M f$. Then the set of function ν_f is countably additive on M.
ii) Lebesgue's monotone convergence theorem.
iii) Fatou's lemma; iv) the theorem on Dominated summability; v) Lebesgue's dominated convergence Theorem.
Necessary & Sufficient condition of Riemann integrability via measure; interrelation between the two modes of integration.
The Concept of L^p-spaces; Inequalities of Holder and Minkowski; Completion of L^p-spaces.
Convergence in Measure, Almost Uniform Convergence, Pointwise Convergence a.e.;
Convergence Diagrams, Counter Examples. Egoroff theorem..
Lebesgue Integral in the Plane. Product σ-algebra. Product Measure. Fubini's Theorem.
Signed Measure and the Hahn Decomposition; The Jordan Decomposition.
The Radon-Nikodym Theorem.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following:

i) Lebesgue measure , Vitali's theorem concerning existence of non-measurable sets,

ii) measurable functions, Theorem relating to non negative μ-measurable function as a limit of a monotonically increasing sequence of non negative simple μ-measurable functions,
iii) Lebesgue's monotone convergence theorem and its applications, Fatou's lemma, Lebesgue's dominated convergence Theorem,
iv) Interrelation between Riemann & Lebesgue integration,
v) Concept of L^p-spaces and its completeness,
vi) Characterizations of Convergence in Measure, Almost Uniform Convergence, Egoroff theorem,
vii) Product Measure. Fubini's Theorem,
viii) Signed Measure and the Hahn Decomposition, Radon-Nikodym Theorem.

Also there is a scope, for applying the acquired knowledge of the above measure- theoretic integration methods/ tools, to solve complex mathematical problems in all of its relevant fields of applications, to develop abstract mathematical thinking as well as in discovering new avenues, that facilitates for higher research and its extensions. It also helps to crack lectureship and fellowship exams approved by UGC & CSIR, GATE and SET.

References :

1. P. R. Halmos, Measure Theory, Von Nostrand, New York, 1950.
2. E. Hewitt & K. Stromberg, Real and abstract Analysis, Third edition, Springer-
3. G. D. Barra, Measure Theory & Integration, Wiley Eastern Limited, 1987.

   verlag,Heidelber & New York, 1975.
4. W. Rudin, Real and Complex Analysis, Tata McGraw- Hill, New York, 1987.
5. I. K. Rana, An introduction to Measure & Integration, Narosa Publishing House, 1997.
6. H. L.Royden, Real Analysis, Macmillan Pub. Co. Inc, New York, 1993.
7. J. F. Randolf, Basic Real and Abstract Analysis, Academic Press, New York, 1968.
8. C. D. Aliprantis and Owen Burkinshaw, Principles of Real Analysis, Academic Press, 2000.
9. K. R. Parthsarathy, introduction to Probaility and Measure, Macmillan Company of India Ltd., Delhi, 1977.
10. R. B. Bartle, Elements of Real Analysis.

Course: 215 123

Functional Analysis : 50 Marks (4 CP) (Duration : 6 Months)

Syllabus :

Metric spaces. Brief discussions of continuity, completeness, compactness. Hölder and Minkowski inequalities (statement only).

Baire's (category) theorem . Banach's fixed point theorem, applications to solutions of certain systems of linear algebraic equations, Fredholm's integral equation of the second kind.

Real and Complex linear spaces. Normed induced metric. Banach spaces, the spaces R^n,C^n, C [a, b],C_{00}, C_0, C and lp($1 \le p \le \infty$). Riesz's lemma. Finite dimensional normed linear spaces and subspaces,completeness, compactness criterion, equivalent norm (with topological significance).

Bounded linear operators, various expressions for its norm. Spaces of bounded linear operators and its completeness. Inverse of an operator. Linear and sublinear functionals, Hahn-Banach theorem for real linear spaces, complex linear spaces and normed linear spaces and some of its simple applications.

Conjugate or Dual spaces . Examples . Separability of the Dual space. Reflexive spaces Examples. weak and weak* convergence. Uniform boundedness principleand its simple applications. The Open mapping Theorem & the Closed graph Theorem .

Innerproduct spaces, Schwarz's inequality, the induced norm. Hilbert spaces, orthogonality, orthonormality, orthogonal complement. The Riesz representation theorem. Bessel's inequality and its generalization. Convergence of series corresponding to orthogonal sequence, Fourier coefficient. Perseval identity. Riesz- Fischer Theorem.

Course Outcomes: On successful completion of this course, students will be able to appreciate how functional analysis uses and unifies ideas from vector spaces, the theory of metrics, and complex analysis. Moreover, students will be able to understand and apply fundamental theorems from the theory of normed and Banach spaces, Hilbert spaces.

References :

1. Rudin, Functional Analysis.
2. Bachman & Narici, Functional Analysis.
3. Kryszic, Functional Analysis .
4. G. F. Simmons, Introduction to topology and modern analysis, McGraw Hill.
5. Dunford.N and J.T.Schwarz, Linear operators, Part 2, Wiley.

6. Taylor.A.E, Introduction to Functional Analysis, Wiley.
7. B.V.Limaye, Functional Analysis, Second Edition, New Age – International limited, Madras.

Course: 215 124

Differential Eq. & Generalized functions : 50 Marks (4 CP) (Duration : 6 Months)

Syllabus :

Differential Equations: 30 marks

Uniqueness and existence of solution of the first order initial value problem .Cauchy Peano existence theorem. Lipschitz condition. Picard's method of successive approximations. Picard- Lindeloeff theorem. Continuation of solution. Dependence on parameters and on initial value.

Linear first order systems. Existence and uniqueness of solutions. Fundamental solutions.

The linear ODE of nth order. Existence and uniqueness of solutions. Fundamental solutions and Wronskian. Properies of the Wronskian.The method of variation of parameters.

Boundary value problems of Sturm Liouville type. Solution. Green's function.Properties of Green's Function.

Eigenvalue problems. Properties of eigenvalues and eigenfunctions for a SturmLiouville problem. Fourier expansion in terms of orthonormalised characteristic functions.

Initial value problem and Green's function. Adjoint operators.

Linear ODE in the complex domain. Ordinary points and regular singular points.Series solution. Frobenius' method.Hypergeometric ,Legendre and Bessel equations. Hypergeometric functions.Legendre polynomials. Bessel functions of first kind and second kind.

Generalised Functions : 20 marks

Definition of test functions. Generalised Functions.Regular and Singular. Delta function and Delata convergence. Product and .Derivative od generalized functions. Slow growth generalized functions. Fourier Transforms. Fourier series of periodic generalized functions.

Course Outcomes:

1. Students will learn about existence and uniqueness of solutions and Picard's method of approximation . This can be directly applied for a numerical approximation.
2. Knowledge of the properties of eigenvalues and eigenfunctions will be useful in studying Mathematical physics.
3. An acquaintance with special functions will be useful for students interested in research in continuum mechanics or theoretical physics.
4. Introductory ideas of phase plane analysis and stability can be utilised by students while studying dynamical systems or mathematical biology.
5. Students will be able to solve/analyse odes arising in different areas of physics.

References :

1. Theory of Ordinary Differential Equations, Coddington and Levinson.
2. Ordinary Differential Equations, Birkoff and Rota.
3. Ordinary Differential Equations, Ince.
4. Generalised Functions in Mathematival Physics, V.S. Vladimiroff.
5. Methods of Mathematical Physics, R. Courant and D. Hilbert.
6. Ordinary Differential Equations, Ross.

Course: 215 125

Operation Research & Numerical Analysis : 50 Marks (4 CP) (Duration : 6 Months)

Syllabus :

Operations Research-25 Marks

Linear Programming: A brief review of Simplex Algorithm, Revised Simplex Method, Dual Simplex Method, Computational procedure.

Nonlinear Programming : Local and Global Minima, Gradient vector, Saddle point Problem, Kuhn-Tucker Conditions of Optimality; Quadratic Programming, Methods due to Wolfe and Beale.

Dynamic Programming : Bellman's Principle of Optimality, Recursive equation approach, Single additive constraint-Additively separable return, Single multiplicative constraint- Additively separable return, Single additive constraint and multiplicatively separable return, Solution of LPP by Dynamic Programming.

Integer Programming : Gomory's all Integer Programming Method , Branch and Bound Method.

Inventory Control : Reasons for carrying inventories, Inventory decisions, Concept of EOQ, Problem of EOQ with no Shortages and with Shortages.

References:

1. H.A.Taha- Operations Research, MacMillan Publ., 1982.
2. Kanti Swarup, P.K.Gupta & Manmohan- Operations Research, Sultan Chand & Sons, New Delhi.
3. S.S.Rao-Optimization Theory and Application, Wiley Eastern Limited., New Delhi.
4. G. Hadley- Nonlinear and Dynamic Programming, Addison-Wesley, 1972.
5. F.S.Hillier & G.J.Lieberman- Introduction to Operations Research, McGraw Hill International Edition.

Numerical Analysis -25 marks

Control of round-off-errors, loss of significance, condition and instability.

Solution of Nonlinear Equations: Iterative methods, convergence of methods, Non-Linear Systems of Equations - Newton's method, Quasi-Newton's method. Roots of Real Polynomial Equations - Bairstow's method of quadratic factors, Graeffe's root squaring method.

Numerical Solution of System of Linear Equations**:** Triangular factorization methods, Matrix inversion method, Ill conditioned matrix, Power method for extreme eigenvalues and related eigenvectors.

Polynomial Interpolation: Hemite interpolation,Uniqueness, Error term, piecewise polynomial interpolation, Cubic spline interpolation.

Approximation of Functions: Weirstrass's approximation theorem (Statement only), Least squares polynomial approximation, Approximation with orthogonal polynomials, Chebyshev polynomials, Uniform approximation.

Richardson extrapolation, Romberg integration, Runge-Kutta methods, Multistep predictor-corrector methods – Milne's method, Adams-Bashforth method, Adams-Moulton method, Convergence and Stability of numerical methods, finite difference methods for BVPs.

Course Outcomes: After completion of the course, the student is expected to :

- solve nonlinear programming problems using Lagrange multiplier, Kuhn-Tucker conditions, Wolfe's and Beale's method,
- find optimal solution of dynamic programming problem,
- learn theory of sequencing models and inventory control and their applications,
- understand basic theories of numerical analysis,
- formulate and solve numerically problems from different branches of science,
- grow insight on computational procedures.

References :

1. S.D. Conte and C. DeBoor, Elementary Numerical Analysis: An Algorithmic Approach, McGraw Hill, N.Y., 1980.
2. K.E. Atkinson, An Introduction to Numerical Analysis, John Wiley and Sons, 1989.
3. Jain, Iyengar and Jain- Numerical methods for scientific and Engineering Computation, New Age International Pub.
4. F.B.Hilderbrand- Introduction to Numerical Analysis, Dover Publication.

Course: 215 127

Numerical Analysis with Computer Application: 25 Marks (2 CP) (Duration : 6 Months) (Practical)

Syllabus :

Numerical Solution of a System of Equations:

Gauss Elimination and Gauss- Seidel Method for a System of Linear Equations

Computation for Finding Inverse Matrix:

L-U Decomposition due to Crout.

Numerical Solution of Algebraic and Transcendental equations :

Newton-Raphson method and Regula -Falsi Method.

Methods for Finding Eigen Pair of a Matrix:

Power Method for Numerically Largest Eigen Value and Corresponding Eigen Vector of a Matrix.

Quadrature Rules:

Simpson's one-third rule.

Numerical Solution of ODE:

Runge- Kutta Method , Taylor Series Expansion Method, Euler's Formula, Picard's Formula, Milne's Formula.

Numerical Solution of PDE:

Finite Difference Method for PDE – Elliptic Type PDE, Parabolic Type PDE, Hyperbolic Type PDE, Crank- Nicholson Scheme for Parabolic Type PDE.

Course Outcomes: At the end of this course a student should be able to :

- solve different type of numerical problems,
- understand better relevant theoretical concepts,
- apply programming skills in interdisciplinary areas such as biological system, physical system etc.,
- analyze data set of various size and interpret outcomes helping her/him to compete in the industry.

References :

1. Computing methods - Berzin and Zhidnov.
2. A first course in Numerical Analysis - Ralston and Rabinowitz.
3. Numerical solution of differential equations - M.K.Jain.
4. Numerical solution of partial differential equations- G.D.Smith.
5. Theory and Problems of Numerical Analysis - F. Scheid

Semester III

<u>Course: 215 231</u>

PDE & Integral Transform. 50 Marks (4 CP) (Duration : 6 Months)

Syllabus :

Partial Differential Equation : 25 Marks (2 CP)

First order partial differential equations - quasi-linear and nonlinear equations. Higher order equations and characteristics. Classification of second order equations. Wave equation – method of separation of variables , Riemann's method, Laplace equation - Green's method. Heat Conduction equations -Boundary value problems. Maximum -minimum principles . Duhamel's principle.

References :

1. Alan Jeffrey , Applied Partial Differential Equations: An Introduction.
2. Peter V. O'Neil, Beginning Partial Differential Equations by John Wiley & Sons, 2nd edition.
3. I.N. Sneddon, Elements of partial differential equations, McGraw Hill, 1986.
4. H.F. Weinberger, A first course in partial differential equations, Blaisdell, 1965.
5. C.R. Chester, Techniques in partial differential equations, McGraw Hill, New York, 1971.
6. K.S. Rao, Introduction to partial differential equations, Prentice Hall, New Delhi, 1997.
7. A. Sommerfeld, Partial differential equations in physics, Academic Press, New York, 1967.
8. V. Vladimirov, Equations of mathematical physics. Dekker, New York, 1971.
9. I. Stakgold, Green's functions and boundary value problems, Wiley, New York, 1979.

Integral Transforms : 25 Marks (2 CP)

The Fourier Transform: Fourier series, properties of Fourier transform, Inversion formula of Fourier transform, Convolution, Translation, Modulation.Transform of derivatives, Perseval formula, Multiple Fourier transforms, Finite Fourier transform, Application to solving boundary valu linear ordinary and partial differential equation.

The Laplace transform:
Definition and Properties of Laplace transform, Initial value theorem, Final value theorem, Heaviside expansion theorem, Transform of derivatives. The inversion theorem, Evaluation of inverse transforms by residue. Application to solving P.D.E., Integral equation, etc.
The Z-Transform:
Properties of the region convergence of the Z-transform. Inverse Z-transform for discrete-time systems and signals, Signal processing and linear system.

Course Outcomes: : After completion of the course, the student is expected to :

- learn to solve different types of PDE,
- apply PDE to problems of geometry and physics,
- to learn theory and properties of Fourier transform, Laplace Transform and Z-Transform and their applications to relevant problems.

References :

1. I.N. Sneddon, Fourier Transform, McGraw Hill, 1951.
2. D. Loknath, Integral Transforms and them Application, C.R.C. Press, 1995.
3. E.J. Watson, Laplace Transforms and Application, Van Nostland Reinhold Co. Ltd., 1981.
4. R.V. Churchill, Operational Mathematics, McGraw Hill, 1958.

Course: 215 232

DIFFERENTIAL GEOMETRY: 50 Marks (4 CP) (Duration : 6 Months)

Syllabus :

Differential manifolds, smooth maps and diffeomorphisms, derivatives of smooth maps, Immersion and submersions, submanifolds, quotient manifolds, Lie groups.

Vector fields, Lie derivative, tangent bundles, vector bundles, flows and exponential maps, Exterior algebra, differential forms, orientable manifolds.

Riemannian manifolds, Curvature and parallel transports, geodesic and geodesic completeness, Hopf-Rinow theorem.

Course Outcomes: On completion of this course, the students will be able to identify, analyze, classify, demonstrate and explain the acquired knowledge mainly on the following :

i) tangent and cotangent spaces; submanifolds,
ii) vector fields and their flows; the Frobenius Theorem,
iii) multilinear algebra, differential forms, the Lie derivative,
iv) Lie groups and Lie derivatives,
v) Riemannian manifolds, geodesic completeness, Hopf-Rinow theorem.

References :

1. Topology and Geometry – Glen E. Bredon, Springer (GTM), 2005
2. Calculus on Manifolds – Michael Spivak, W.A. Benjamin, 1965
3. Foundations of differentiable manifolds and lie groups – F. W. Warner, Springer (GTM)
4. Differential Geometry, Lie groups and symmetric spaces – S. Helgason, Springer (GSM)
5. Differential Topology – V. Guillemin & A. Pollock, Englewood & Cliffs
6. Elementary Differential Topology – J. R. Munkres, Princeton University Press, 1963
7. Lecture Notes on Elementary Topology and Geometry – I. M. Singer and J. A. Thorpe, Springer (UGTM)
8. A course in Differential Geometry and Lie Groups – S. Kumeresan, Hindustan Book Agency (TRIM 22)
9. Topics in Differential Topology – A. Mukherjee, Hindustan Book Agency (TRIM 34)

Course: 215 233

Theory of computations & Graph Theory : 50 marks (4 CP) (Duration : 6 Months)

Syllabus :

Theory of Computation : 25 marks (2 CP)

Finite Automata – Determinstic and Non-Deterministic ,ε-moves – Elimination and Uses of ε-moves, NFA with ε-moves, ε-closures, Equivalence of NFA and DFA,, Regular Expressions, Regular Languages.

Context Free Grammers and Language, Pushdown Automata, Turing Machines.

References :

1. The Theory of Computation – Bernard M Moret
2. Theory of Computer Science - K L P Mishra, N Chandrasekharan
3. Introduction to Automata Theory, Languages and Computation - John E Hopcroft, Rajeev Motwani, Jeffery D Ullman

Graph Theory : 25 marks (2 CP)

Graphs and digraphs, Geometrical representation of graphs, Simple graphs, Null graphs, and regular graphs, degree of a vertex and degree- sequence of a graph. Handshaking lemma due to Euler and some basic properties of a graph. In - degree and out - degree of a vertex in a digraph. Simple digraph and underlying graph. Representation of binary relations on finite sets by digraphs. Reflexive, symmetric and transitive digraphs.

Sub graph, spanning sub graph, induced sub graph on a vertex set and induced sub graph on an edge set. Isomorphism of graphs. Walks, paths, circuits and cycles with their properties, concatenation of two walks.

Connected and disconnected graphs. A necessary and sufficient condition for a graph to be disconnected. Component of a graph, decomposition of a graph into finite number of components, acyclic graph and cycle edge of a graph. Some properties of connected graphs. Complete graphs, disconnecting sets, cut sets, bridge, separating sets and cut vertices, distance between two vertices of a graph. Complement of a graph, Self complementary graphs, Ramsey problem. Bipartite graphs, complete bipartite graphs, necessary and sufficient condition for a simple graph to be bipartite.

Eulerian and Hamiltonian graphs: Euler trials, Euler circuits, Edge traceable graphs, Euler graphs, Euler's Theorem. Fleury's algorithm, Konigsberg bridge problem. Hamiltonian path, Hamiltonian cycle, Hamiltonian graph. A necessary condition for the existence of a Hamiltonian cycle in a connected graph. Sufficient condition for a simple connected graph to be Hamiltonian. Dirac's Theorem, Ore's Theorem(statement only) and its use.

Trees and forests with their properties. Minimally connected graphs, spanning trees. weighted graphs, weight of a spanning tree and minimal spanning trees, Kruskal's algorithm for a minimal spanning tree. The shortest path problem, traveling salesman problem.

Matrix representation of graphs.

Course Outcomes: After completing this course, the student will be able to learn mathematical foundations of computation including automata, the theory of formal languages and grammars. The students will be able to apply principles and concepts of graph theory in practical situations such as computer science, physical and engineering sciences.

References:

1. N. Deo, Graph Theory with Applications to Engineering and Computer Science, Prentice Hall of India, 2000.
2. C. L. Liu, Elements of Discrete Mathematics, McGraw-Hill Book Co.
3. J. P. Tremblay & R. Manohar, Discrete Mathematical Structures with Applications to Computer Science, Mcgrow-Hill Book Co., 1997.
4. F. Harary, Graph Theory, Addison-Wesley Publishing Company, 1972.
5. J. Gross & J. Yellen, Graph Theory and its Applications, CRC Press (USA), 1999.

Course: 215 234

Minor Elective I

Group A

1. Operator Theory & Banach Algebra : 50 Marks, 4 CP (Duration : 6 Months)

Dual spaces, Representation Theorem for bounded Linear functionals on C[a,b] and L_p spaces,Dual of C[a,b] & L_p spaces, weak & weak* convergence, Reflexive spaces.

Bounded Linear Operators, Uniqueness Theorem, Adjoint of an Operator and its Properties; Normal, Self Adjoint, Unitary, Projection Operators, their Characterizations & Properties. Orthogonal Projections, Characterizations of Orthogonal Projections among all the Projections. Norm of Self Adjoint Operators, Sum & Product of Projections, Invariant Subspaces.

Spectrum of an Operator, Finite Dimensional Spectral Theorem, Spectrum of Compact Operators, Spectral Theorem for Compact Self Adjoint Operators (statement only)

Properties of Bounded Linear Operators, Existence and Representation of the Inverse of I—T, Representation of the Resolvent Operators, Non- emptiness of the Resolvent Set, Spectral Radius and its Representation, Spectral Mapping Theorem for Polynomials.

Banach Algebra, Banach Sub Algebra, Identity element, invertible elements, resolvent set and resolvent operator, analytic property of resolvent operator, compactness of spectrum. Division Algebra, Gelfand-Mazur Theorem. Topological divisors of zero. Spectral radius, spectral mapping theorem for polynomial, formula for spectral radius. Complex homomorphism, Gleason-Kahane-Zalazko Theorem, Commutative Banach Algebra, Ideals, maximal ideals, Quotient space as a Banach Algebra under certain conditions. Gelfand theory on representation of Banach Algebra, Gelfand transform, weak Topologies, weak-* Topology, Gelfand Topology, Banach Alaoglu Theorem.

References:

1. Rudin, Functional Analysis
2. Schaffer, Topological Vector Spaces
3. Bachman & Narici, Functional Analysis

4. Kryszic, Functional Analysis
5. Diestal, Applications of Geometry of Banach Spaces
6. Horvat, Linear Topological spaces

2. Number Theory: 50 Marks, 4 CP (Duration : 6 Months)

Primes and factorization. Division algorithm. Congruence and Modular arithmetic. Chinese remainder theorem. Euler phi function. Primitive roots of Unity, existence and number of primitive roots. Quadratic residues. The lawsof Quadratic reciprocity law and its applications. The Jacobi symbol and its properties.Arithmetical function & Dirrichlet multiplication. Mobius inversion formula. Pell's equation and some other diphantine equations.

Riemann zeta function, functional equation, prime number theorem. Mobius inversion formula.Algebraic numberfields and their rings of integers.

References:

1. D. M. Burton, Elementary number theory, Wm. C. Brown Publishers, Dubuque , Iowa, 1989.
2. G. A . Jones and J.M. Jones, Elementary number theory, Springer, 1998
3. I. Niven, S.H.Zuckerman and L. H. Montgomery, An Introduction to the theory of Numbers, John Wiley, 1991
4. T. M. Apostol, An Introduction to Analytic Number theory, Narosa Publishing House, 1980.
5. S. Lang, Algebraic Number theory, GTM Vol. 110, Springer- Verlag, 1994.

3. Lie Groups & Lie Algebras: 50 Marks, 4 CP(Duration : 6 Months)

1. Topological groups and Lie groups, exponential map.
2. Lie algebras, structure theorems for Lie algebras (Killing form, roots systems, Weyl groups,Poincaré-Birkoff-Witt).
3. Haar Measure.
4. Linear representations of Lie algebras.
5. Applications to classical groups, closed subgroups of GL(n,R).
6. Semi-simple Lie groups
7. Symmetric spaces
8. Algebraic groups
9. Affine Lie algebras, Kac-Moody algebras,

References:

1. V.G. Kac, Infinite dimensional Lie algebras, Cambridge University Press, Third Edition
2. J. Fuchs, C. Schweigert, Symmetries, Lie algebras and Representations : A Gra duate Course forPhysicists, Cambridge Monographs on Mathematical Physics
3. M. Wakimoto, Lectures on Infinite Dimensional Lie Algebras, World Scientific
4. U. Ray, Automorphic forms and Lie superalgebras, Springer
5. M. Raghunathan : Discrete subgroups of Lie groups, Springer (1972)
6. R. Zimmer : Ergodic theory and semisimple groups,Birkhauser (1984)
7. G. Margulis : Discrete subgroups of semisimple Lie groups, Springer (1991)
8. D. Witte-Morris : Ratner's theorems on unipotent flows, Chicago LMS(2004)
9. Y. Benoist: Five lectures on lattices, SMF (2007)
10. R. Carter : Lie algebras of finite and affine type, Cambridge 2005
11. J. Humphreys : Introduction to Lie algebras and representation theorem

Course: 215 234

Minor Elective I

Group B

1. **Ordinary Differential Equations and Dynamical Systems**: **50 Marks, 4 CP** (**Duration : 6 months)**

Second Order ODE in the phase plane.Autonomous equations in the phase plane,the damped linear oscillator,limit cycles.Plane autonomous systems and linearization: linear approximation of equilibrium points,general solution of linear autonomous plane systems.Phase paths of linear autonomous systems.Phase diagrams.Geometrical aspects.

Index of an equilibrium point.The index at infinity.the phase diagram at infinity.Homoclinic and heteroclinic paths.

Averaging methods. Energy balance method for limit cycles. Amplitude and frequency estimates. Nearly- periodic solutions. Periodic solutions and Harmonic balance method.

Stability.Poincare and Lyapunov stability.Solutions and paths,linear systems,zero solutions of nearly linear systems. Lyapunov functions.The existence of periodic solutions. The Poincare Bendixson theorem.

References :

1. Nonlinear Ordinary Differential Equations, *D W Jordan, P Smith.*
2. Nonlinear Differential Equations and Dynamical Systems, *F Verhulst.*

2. Continuum Mechanics * : 50 marks (4 CP) (Duration : 6 Months)

Introduction .Idea of continua.Deformation and motion.Axiom of continuity.Continuum motion as a real continuous map.

Analysis of strain.Eulerian and Lagrangian reference frames.Cartesian tensors.Deformation tensor ,strain tensor.Physical interpretation.Displacement and strain.Principal strains.Cauchy strain quadric. Infinitesimal strain and rotation..Compatibility equations.

Analysis of stress. Body forces. Surface forces. Stress vector. Stress tensor.

Balance laws. Equation of continuity. Equation of balance of linear momentum. Stress equations of motion. Equations of balance of angular momentum. Equation of balance of Energy (laws of thermodynamics).

Constitutive equations. Generalized Hooke's law for a solid body. Rate of strain for fluid.

Linear isotropic solids and fluids. Physical interpretation of parameters λ and μ .Equation of motion.

Incompressible viscous fluids. Steady state motion. Equation of motion.

References :

1. Continuum Mechanics, Y.C. Fung.
2. Mechanics of Continum, A.C. Eringen.
3. Mathematical Theory of Elasticity, I. S. Sokolnikoff.
4. Treatise on Hydrodynamics, Ramsey and Besant.
5. Theoretical Hydrodynamics, Kochin , Kebel and Roze.

3. Inegral Equations and Boundary Value Problems: 50 marks (4 CP) (Duration : 6 Months)

Integral Equations :

Definition of Integral Equation and their classification; Reduction of differential equation to an integral equation; Eigen values and Eigen functions; Fredholm Integral Equations of second kind with separable kernels, Reduction to a system of algebraic equation.

Method of successive approximation, Neumann series; Existence and uniqueness of the solution of Fredholm equation and Volterra equation. Resolvent kernel and its result, Application to iterative scheme to Fredholm and Volterra equation.

Classical Fredholm theory, Fredholm theorems.

Symmetric kernels, Complex Hilbert Space, Orthonormal system of functions, Fundamental properties of eigen values and eigen functions, Hilbert Schmidt theorem.

Singular Integral equation, Solution of Abel's Integral equation. Solution of Volterra equation of convolution type by Laplace transform.

Boundary Value Problems :

Definition of Boundary Value Problem for an ordinary differential equation of the second order and its reduction to a Fredholm integral equation. Green’s function approach to reduce boundary value problems to integral equation forms. Integral equation formulations of boundary value problems with more general and inhomogeneous boundary conditions.

Integral representation for the solution of the Laplace’s and Poisson’s equations. Newtonian single-layer and double layer potentials. Integral equation formulation of boundary value problems for Laplace’s equation. Poisson’s integral formula.

Perturbation techniques and its applications to mixed boundary value problems. Two part and three part boundary value problems. Solutions of electrostatic problems involving a charged circular and annular disc, a spherical cap, an annular spherical cap in a free space or a boundary space.

References :

1. R. P. Kanwal- Linear Integral Equation – Theory and Techniques, Academic Press.
2. W. V. Lovitt- Linear Integral Equations, Dover, New York.
3. F. G. Tricomi- Integral Equations, Dover.
4. S. G. Mikhlin- Integral Equations, Pergamon Press, Oxford.
5. I. N. Sneddon-Mixed boundary value problems in potential theory, North Holland, 1966.
6. I. Stakgold- Boundary value problems of mathematical physics, Vol-I and II, Macmillan, 1969.

Course: 215 236

Mathematical Modeling (Practical) : 25 marks (2 CP)

Syllabus :

Basic Principles : Modelling using difference/ differential equations. Models in Mathematical ecology- Single species and two species models, Traffic Flow Model.

Examples of Mathematical Modelling from Mechanics.

Stochastic Models: Birth and Death Processes.

Course Outcomes: After completing this course, the student will be able to create mathematical models in different domains such as the physical, natural, biological science and learn different techniques to solve the problems.

References :

1. Mathematical Modelling --J. N. Kapur.
2. The Nature of Mathematical Modelling -- Neil Gershenfeld (CUP).

Semester : IV

Courses: 215 241 & 215242

Major Elective I & Major Elective II

1. Advanced Topology-I Marks : 50 (4 CP)(Duration : 6 Months)

Nets and Filters : Inadequacy of sequences. Nets & filters. Topology and convergence of nets & filters. Subnets. Ultranets & Ultra filters. Canonical way of converting nets to filters and vice-versa. Chracterizations of Hausdorffness , compactness and continuity in terms of nets and filters .Convergence of nets and filters in product spaces.

Identification topology and Qutient spaces.

Local Connectedness, Path- connectedness, Total disconnectedness, Zero -dimensional spaces, Extremally disconnected spaces.

Local Compactness and One Point Compactification. Stone- Cech Compactification.

Embedding and Metrization. Embedding Lemma and Tychonoff Embedding. The Urysohn Metrization Theorem. The Nagata – Smirnov Metrzation Theorem.

Paracompacntess : Different types of refinements and their relationships. Paracompacntess – definition in terms of locally finite refinement, various characterizations and examples. A. H. Stone's Theorem concerning paracompactness of metric spaces. Partition of unity and Paracompacntess. Properties of Paracompacntess w.r.to subspaces and products.

Uniform spaces : Definition and examples. Base and subbase of a uniformity . Uniform topology, uniform continuity and product uniformity. Uniformization of topological spaces. Uniform property. Uniformity generated by a family of pseuometrics. Cauchy filter. Completeness of uniform spaces. Completion of uniform spaces. Compactness and uniformity. Uniform cover.

References :

1. General Topology by J. L. Kelley, Van Nostrand.
2. General Topology by S. Willard, Addison-Wesley.
3. Topology by J. Dugundji, Allyn and Bacon.
4. Topology, A first course by J. Munkres, Prentice Hall, India.

5. Introduction to topology and modern analysis by G. F. Simmons, McGraw Hill.
6. Introduction to General topology by K. D. Joshi, Wiley Eastern Ltd.
7. General Topology by Engelking, Polish Scientific Publishers, Warszawa.
8. Counter examples in Topology by L. Steen and J. Seebach.

2. Advanced Topology-II Marks : 50 (4 CP)(Duration : 6 Months)

Algebraic Topology : Marks 30

Covering spaces and covering maps – properties and examples, Path Lifting and Monodromy theorems, Deck transformation, Van Kampen's theorem (with a discussion of free and amalgamated products of groups), computing fundamental groups via covering spaces.

Singular Homology - Chain complex, Homotopy invariance of a homology, Relation between π_1 and H_1, Relative homology, Exact homology sequence, Excision theorem, Universal coefficient theorem, Kunneth formula, Mayer-Vietoris sequence, Eilenberg-Steenrod axioms for abstract homology theory, Construction of spaces - spherical complexes, cell complexes and more adjunction spaces; Idea of Cohomology and cup product.

References :

1. W. S. Massey – Algebraic Topology
2. W. S. Massey – Singular Homology Theory
3. E. H. Spanier – Algebraic Topology
4. B. Gray – Homotopy Theory An Introduction to Algebraic Topology
5. C. R. Bredon – Geometry and Topology

Rings of Continuous Functions : Marks 20

The ring C(X) & its subring $C^*(X)$, their Lattice structure. Ring homomorphism and lattice homomorphism.

Zero-sets, cozero-sets, completely separated sets and its characterization, C-embedding & C^*-embedding and their relation, Urysohn's extension theorem . Characterizations of Normal spaces and Pseudocompact spaces in terms of C-embedding & C^*-embedding.

Ideals, maximal ideals, prime ideals, Z- ideals; Z-filters, Z- ultrafilters, prime filters and their relations.

Convergence of Z – filters, cluster points, prime Z – filters and convergence and fixed Z-filters

Completely regular spaces and the zero-sets, weak topologies determined by C(X) and C^*(X). Stone-Čech's therem concerning adequacy of Tychonoff spaces X for investigation of C(X) and C^*(X).

Fixed ideals and compactness, fixed maximal ideals of C (X) and C^*(X), their characterizations. Structure spaces.

References:

1. Richard E. Chandler, Hausdorff Compactifications (Marcel Dekker, Inc. 1976).
2. L. Gillmen and M. Jerison, Rings of Continuous Functions (Von Nostrand, 1960).
3. Topological Structures (Halt Reinhurt and Winston, 1966).

3. ADVANCED COMPLEX ANALYSIS : 50 (4 CP) (Duration : 6 Months

Harmonic functions, Characterisation of Harmonic functions by mean-valueproperty. Poisson's integral formula. Dirichlet problem for a disc.Doubly periodic functions. Weierstrass Elliptic function, Weierstrass σ-functionand their properties.

Entire functions, M(r, t) and its properties (statements only). Meromorphicfunctions. Expansions. Definition of the functions m (r, a), N(r, a) and T(r).Nevanlinna's first fundamental theorem. Cartan's identity and convexity theorems.

Orders of growth. Order of a meromorphic function. Comparative growth of logM(r)and T(r). Nevanlinna's second fundamental theorem. Estimation of S(r) (Statementonly). Nevanlinna's theorem on deficient functions. Nevanlinna's five-pointuniqueness theorem. Milloux theorem. Milloux basic result. Idea of fix points. (25)

Functions of several complex variables. Power series in several complex variables.Region of convergence of power series. Associated radii of convergence. Analyticfunctions. Cauchy-Riemann equations. Cauchy's integral formula. Taylor'sexpansion. Cauchy's inequalities. Zeros and Singularities of analytic functions.

Maximum modulus theorem. Weierstrass preparation theorem.

References :

1. E. T. Copson **:** An Introduction to the Theory of Functions of a Complex Variable.
2. E. C. Titthmarsh **:** The Theory of Functions.
3. A. I. Markushevich **:** Theory of Functions of a Complex Variable (Vol. I, II &III).
4. L. V. Ahlfors **:** Complex Analysis.
5. J. B. Conway **:** Functions of One Complex Variable.

6. A. I. Markushevich **:** The Theory of Analytic Functions, A Brief Course.
7. G. Valiron **:** Integral Functions.
8. C. Caratheodory **:** Theory of Functions of a Complex Variable.
9. R. P. Boas **:** Entire Functions.

4. Advanced Functional Analysis - 50 marks (4 CP) (Duration : 6 Months)

Topological Vector Spaces, Local base and its properties, Separation properties, Locally compact topological vector space and its dimension. Convex hull and representation theorem, Extreme points, Symmetric sets, Balanced sets, Absorbing sets, Bounded sets in topological vector space. Linear operators over topological vector space, Boundedness and continuity of linear operators, Minkowski functionals, Hyperplanes, Separation of convex sets by Hyperplanes, Krein-Milman Theorem on extreme points.

Geometric form of Hahn Banach Theorem . Uniform- boundedness principle, open mapping theorem and closed graph theorem for Frechet spaces. Banach-Alaoglu theorem.

Locally convex topological vector spaces, Criterion for normability, Seminorms, Generating family of seminorms in locally convex topological vector spaces.

Approximation Theory in Normed Linear space, Best approximation, Uniqueness Criterion, Separable Hilbert Space, Strict convexity of Hilbert space.

Algebra and some properties of the space C(X), Stone-Weierstrass Theorem.

References:

1. Rudin, Functional Analysis
2. Schaffer, Topological Vector Spaces
3. Bachman & Narici, Functional Analysis
4. Kryszic, Functional Analysis
5. Narici-Beckerstein, Topological Vector Space

5. Advanced Real Analysis : 50 marks (4 CP) (Duration : 6 Months)

Representation of real numbers by series of radix fractions. Sets of real numbers,Derivatives of a set. Points of condensation of a set. Structure of a bounded closed set.

Perfect sets. Perfect kernel of a closed set. Cantor's nondense perfect set. Sets of first and second categories, residual sets. Baire one functions and their basic properties. One-sided upper and lower limits ofa function. Semicontinuous functions. Dini derivates of a function. Zygmund's monotonicity criterion.

Vitali's covering theorem. Differentiability of monotone functions and of functions of bounded variation. Absolutely continuous functions, Lusin's condition ,characterization of AC functions in terms of VB functions and Lusin's condition. Concepts of VB*, AC*,VBG*, ACG* etc. functions. Characterization of indefinite Lebesgue integral as an absolutely continuous function.

Generalized Integrals **:** Gauge function. Cousin's lemma. Role of gauge functionin elementary real analysis. Definition of the Henstock integral and its fundamental

properties. Reconstruction of primitive function. Cauchy criterion for Henstock integrability. Saks-Henstock Lemma. The Absolute Henstock Integral. The McShane integral. Equivalence of the McShane integral, the absolute Henstock integral and the Lebesgue integral. Monotone and Dominated convergence theorems. The Controlledconvergence theorem.

Definition and elementary properties of the Perron integral and its equivalence with the Henstock integral. Definition of the (special) Denjoy integral and its equivalence with the Henstock integral (characterization of indefinite Henstock integral as a continuous ACG*function). Density of arbitrary sets. Approximate continuity. Approximate derivative.

References :

1. E. W. Hobson **:** The Theory of Functions of a Real Variable (Vol. I and II).

2. I. P. Natanson **:** Theory of Functions of a Real Variable (Vol. I and II).

3. R. Henstock **:** Lectures on the Theory of Integration.

4. E. J. McShane **:** Unified Integration.

5. S. Saks **:** Theory of the Integral.

6. HARMONIC ANALYSIS : 50 marks (4 CP) (Duration : 6 Months)

FOURIER ANALYSIS: Fourier series, Pointwise and uniform converges of Fourier series, Fourier transforms, Riemann-Lebesgue lemma, Inversion theorem, Parseval Identity, Plancharel theorem.

ELEMENTS OF BANACH ALGEBRAS: Analytic properties of functions from C to Banach algebras, Spectrum and its compactness, commuta tive banach algebra, Maximal Ideal space, Gelfand Topology.

TOPOLOGICAL GROUPS: Definition, Basic properties, subgroups, quotient groups, locally compact topological groups, examples.

HAAR INTEGRAL: Haar measure, its existence and uniqueness on locally compact topological group, Haar integral, Examples of Haar measures, character.

GENERALIZATION OF FOURIER TRANSFORM : Fourier trasnform on L'(G) and L (G) (G being a locally compact topological group) Positive definite functions, Bochner characterization, Inversionformula, Plancheral theorem, Pontrjagin Duality theorem.

REFERENCES:

1. Bachman G., Elements of Harmonic Analysis.

2. Henry Helson, Harmonic Analysis, Hindustan Book Agency.

7.Representation Theory of Groups : 50 marks (4 CP) (Duration : 6 Months)

Representations, Subrepresentations, Tensor products, Symmetric and Alternating Squares.

Characters, Schur's lemma, Orthogonality relations, Decomposition of regular represent-ation, Number of irreducible representations, canonical decomposition and explicit decompositions. Subgroups, Product groups, Abelian groups. Induced representations.

Examples: Cyclic groups, alternating and symmetric groups.

Integrality properties of characters, Burnside's $p^a q^b$ theorem. The character of induced representation, Frobenius Reciprocity Theorem, Meckey's irreducibility criterion, Examples of induced representations, Representations of super solvable groups.

References :

1. M. Burrow, Representation Theory of Finite Groups, Academic Press, 1965.
2. N. Jacobson, Basic Algebra II, Hindustan Publishing Corproation, 1983.
3. S. Lang, Algebra, 3rd ed. Springer (India) 2004.
4. J.P. Serre, Linear Representation of Groups, Springer-Verlag, 1977.
5. Walter Ledermann, Introduction to Group Characters, Cambridge University Press.

8.Fuzzy sets & Their applications **: 50 marks (4 CP) (Duration : 6 Months)**

Unit: 1
Fuzzy sets-Basic definitions. Level sets, Convex fuzzy sets. Basic operations on fuzzy sets.Types of fuzzy sets. Cartesian products. Algebraic products bounded sum and difference f norms and t-co norms.

Unit: 2
The Extension principle-The Zadeh's extension principle image and inverse image of fuzzy sets

Unit: 3
Fuzzy numbers. Elements of fuzzy arithmetic., Fuzzy Relations and Fuzzy Graphs-fuzzyrelations on fuzzy sets. Composition of fuzzy relations, Min-Max composition, and itsproperties.

Unit: 4
Fuzzy compatibility relations Fuzzy relation equations. Fuzzy graphs. Similarity relation.

Unit : 5
Fuzzy Logic- An overview of classical logics. Multivalued logics. Fuzzy. Propositions.
Fuzzy quantifiers. Linguistic variables and hedges.

Unit: 6
Possibility Theory-Fuzzy measures. Evidence theory, Necessity; measure. Possibility theory versus probability theory.

Unit : 7
Decision Making in Fuzzy Environment -individual decision-making. Multiperson decision making. Multicriteria decision-making. Multistage decision making fuzzy ranking methods.Fuzzy linear programming.

References :

1. George J Klir and Tina A Folger, Fuzzy sets<Uncertainty and Information, Prentice Hall of India, 1988.
2. H. J. Zimmerman, Fuzzy Set theory and its Applications, 4th Edition, Kluwer Academic Publishers, 2001.

3. George J Klir and Bo Yuan, Fuzzy sets and Fuzzy logic: Theory and Applications, Prentice Hall of India, 1997.
4. Timothy J Ross, Fuzzy Logic with Engineering Applications, McGraw Hill International Editions, 1997.
5. Hung T Nguyen and Elbert A Walker: A First Course in Fuzzy Logic, 2nd Edition Chapman & Hall/CRC 1999.
6. Jerry M Mendel, Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions, PH PTR, 2000.
7. John Yen and Reza Langari, Fuzzy Logic: Intelligence, Control and Information, Pearson Education, 1999.

9. Differential Topology : 50 marks (4 CP) (Duration : 6 Months)

Smooth mappings: Inverse Function Theorem, Local Submersion Theorem (Implicit Function Theorem).

Differentiable manifolds: Differentiable manifolds and submanifolds; examples, including surfaces, S^n, RP^n, CP^n and lens spaces; tangent bundles; Sard's Theorem and its applications; differentiable transversality; orientation.

Vector fields and differential forms: Integrating vector fields; degree of a map, Brouwer Fixed Point Theorem, No Retraction Theorem, Poincare-Hopf Theorem; differential forms, Stokes Theorem.

Vector fields and differential forms: Integrating vector fields; degree of a map, Brouwer Fixed Point Theorem, No Retraction Theorem, Poincare-Hopf Theorem; differential forms, Stokes Theorem.

References:

1. Guillemin Pollack, *Differential Topology*, Prentice-Hall, 1974 (basic reference).
2. Hirsch, *Differential Topology,* Springer, 1976.
3. Milnor, *Topology from the Differentiable Viewpoint,* University of Virginia Press, 1965.
4. Spivak, *Calculus on Manifolds*, Benjamin, 1965 (differentiation, Inverse Function Theorem, Stokes Theorem).

10. Funamentals of Computer Science Theory & Practicals: 50 marks (4 CP) (Duration : 6 Months)

Introduction to Complexity Theory (P, NP, NP-hard, NP-complete etc.). Automata Theory and Formal Languages (finite automata, NFA, DFA, regular languages, equivalence of DFA and NFA, minimization of DFA, closure properties of regular languages, regular grammars, context free grammars, parse-trees, Chomsky Normal Form, top-down parsing).

Randomization and Computation (Monte Carlo and Las Vegas algorithms, Role of Markov and Chebyscheff's inequalities, Chernoff bounds in randomized algorithms, applications of probabilistic method).

Special Topics in Theoretical Computer Science, such as Approximation Algorithms, Number Theoretic Algorithms, Logic and Computability.

References

1. G. Ausiello, P. Crescenzi, G. Gambosi, V. Kann, A. Marchetti-Spaccamela, Complexity and Approximation, Springer Verlag, Berlin, 1999.

2. J. Hein, Discrete Structures, Logic and Computatibility, Jones and Barlett, 2002.

3. P. Linz, An Introduction to Formal Languages and Automata, Narosa, New Delhi, 2004

11. Algebraic Geometry: 50 marks (4 CP) (Duration : 6 Months)

Polynomial rings. Hilbert Basis theorem. Noether normalization Lemma.

Hilbert Nullstellensatz.

Elementary daimension theory.

Smoothness. Curves. Divisors on curves.

Bezouts theorem.

Abelian differential. RiemannRoch theorem for curves.

References :

1. W. Fulton, Algebraic Curves : An Introduction to algebraic Geometry, Addision – Wesley Publishing company, 1989.
2. J. Harris, Algebraic Geometry. A first course, GTM, 133. Springer- Verlag, 1995.

3. R. Shafarevich, Basic algebraic geometry, 1. Varieties in projective space, Springer-Verlag, 1994.
4. C. Musili, Algebraic geometry for beginners, Texts and readings in Mathematics, 20. Hindustan Book Agency, 2001.

12. Commutative Algebra : 50 (4 CP) (Duration : 6 Months)

Properties of Maximal, prime and primary ideals, radical, nil-radical and Jacobson radical, local ring, Nakayama's lemma, prime spectrum of a ring and Zariski topology, Noetherian and Artinian rings, Hilbert's Nullstellensatz theorem. Finitely generated modules, tensor product of modules, exactness properties of tensor product.

Rings and Modules of fractions, localization and local properties, primary decomposition and associated primes, Integral dependence and independence, integral closure, integrally closed integral domain, Going up and going-down theorems.

Valuation rings, discrete valuation ring, Dedekind's domain, graded rings and modules, completion of modules, Krull intersection theorem. Dimension theory – Dimension theorem of Noetherian local rings.

References

1. M. F. Atiyah and I. G. MacDonald, *Introduction to Commutative Algebra*, Addison–Wesley, 1969.
2. N. Bourbaki, *Commutative Algebra*, Hermann, 1972.
3. D. Eisenbud, *Commutative Algebra with a View Towards Algebraic Geometry*, Springer–Verlag, 1995.
4. I. Kaplansky, *Commutative Rings*, The University of Chicago Press, 1974.
5. E. Kunz, *Introduction to Commutative Algebra and Algebraic Geometry*, Birkh¨auser, 1985.
6. H. Matsumura, *Commutative Algebra*, Benjamin, 1970.
7. H. Matsumura, *Commutative Ring Theory*, Cambridge University Press, 1986.
8. M. Nagata, *Local Rings*, Wiley Interscience, New York, 1962.
9. O. Zariski and P. Samuel, *Commutative Algebra*, Vol. 1, Van Nostrand, 1958.

13. Non- Commutative Rings: 50 Marks, 4 CP **(Duration : 6 Months)**

Tensor products. Chain Conditions. Semisimplicity & Structure of Semisimple Rings. Wedderburn Artin Theorem. Jacobson Radical. Prime Radical & Prime and Semiprime Rings. Structure of Primitive rings and Density Theorem. Direct Products , Sub Direct Sums & Commutativity Theorems.

Division Rings. Maximal Subfields. Polynomials over Division rings and Local rings. Semi Local rings and Idempotents. Perfect & Semiperfect Rings.

References:

!. T. Y. Lam , Non -Commutative Rings, Springer Verlag, 1991.

2. I. N. Herstein, Non -Commutative Rings, Carus Monographs of AMS, 1968.

3. N . Jacobson, Basic Algebra II, WH Freeman, 1989.

4. Louis H. Rowen, Ring Theory (Student Edition), Academic Press, 1991.

14. Non Linear Differential Equations and Dynamical Systems: 50 Marks, 4 CP **(Duration : 6 Months)**

Second Order ODE in the phase plane.Autonomous equations in the phase plane,the damped linear oscillator,limit cycles.Plane autonomous systems and linearization: linear approximation of equilibrium points,general solution of linear autonomous plane systems.Phase paths of linear autonomous systems.Phase diagrams.Geometrical aspects.

Index of an equilibrium point.The index at infinity.the phase diagram at infinity.Homoclinic and heteroclinic paths.

Averaging methods. Energy balance method for limit cycles. Amplitude and frequency estimates. Nearly- periodic solutions. Periodic solutions and Harmonic balance method. Non-autonomous equations. Lindstedt's method.

Stability.Poincare and Lyapunov stability.Solutions and paths,linear systems,zero solutions of nearly linear systems. Lyapunov functions.

The existence of periodic solutions. The Poincare Bendixson theorem.

Simple bifurcations

Perturbation methods.Direct perturbation of Duffing's equation.Undamped and damped pendulum.Amplitude – phase perturbation. Lindstaedt's method.Singular perturbation.Lighthill's method.

References :

1. D W Jordan, P Smith, Nonlinear Ordinary Differential Equations.

2. F Verhulst, Nonlinear Differential Equations and Dynamical Systems.

15. **Non- Linear Dynamical Systems and Chaos : 50 (4 CP) (Duration : 6 Months)**

(To be offered only with Nonlinear Differential Equations and Dynamical Systems as minor/major elective)

Nonlinear Systems.Bifurcations and Symmetry breaking. – the origin of Bifurcation Theory.Examples of different types of bifurcations.Transcritical, pitchfork,saddle-node.Centre manifolds.Bifurcation of equilibrium solutions and Hopf bifurcation.

Introduction to the thery of Chaos.The Lorenz equations and associated maps.Duffing's equation with negative stiffness.One dimensional chaos.The quadratic map.The tent map.

Bifurcatins in one dimensional maps.Period doubling bifurcations.The Feigenbaum number.Two dimensional maps.Bifurcation in two dimensional maps.

Hamiltonian systems.Recurrence.Periodic solutions.Invariat torus and chaos.

References :

1. Dynamical systems diffential equations,maps and chaotic behavior, Arrowsmith and Place.

2. Chaotic Dynamics, Baker and Gollub.

3. Nonlinear Systems, Drazin.

4. Nonlinear Differential equations and Dynamical Systems, Verhulst.

5. Differential equations and dynamical systems, Guckenheimer and Holmes.

16. Fluid Plasma Theory: 50 Marks, 4 CP (Duration : 6 Months)

Plasma as the fourth state of matter. Thermal ionisation, Saha equation, Basic defining properties of plasma, Debye screening, Plasma parameter, Short range and long range forces. Some elementary ideas of electric and magnetic fields.

Charged particles in electric and magnetic fields**:** Larmor orbits, particle drifts, Adiabatic invariants. Applications. Magnetic mirror. Charged particle motion in electromagnetic waves.

Waves in Plasma**:** Propagation characteristics of plane and waves – dispersion relation. Energy, power-equation of conservation of energy. Positive and negative energy waves.

Langmuir waves, Electromagnetic waves, Ion-acoustic waves. Dielectric tensor. Dispersion relation. Propagation along and perpendicular to the magnetic field. Cut-off and resonance. Wave normal surface. Refractive index surface. Alfvén Wave. Ion-cyclotron wave, electron and ion whistler.

References :

1. F.F. Chen, Introduction to Plasma Physics, Plenum Press, New York and London (1977).
2. T.W. Stix, The Theory of Plasma Waves, McGraw Hill Book Company, New York, San Francisco, Toronto, London (1962).
3. T.J.M. Boyd and J.J. Sanderson, Plasma Dynamics, Nelson, London.
4. W.B. Thompson, An introduction to Plasma Physics, Pergamon Press, Oxford.

17. Kinetic Plasma Theory: 50 Marks, 4 CP (Duration : 6 Months)

Kinetic equations of plasma: Boltzmann equation (non-collisional and collisional), BBGKY hierarchy equations, Vlasov equations.

Macroscopic Equation of Plasma: Macroscopic equations, fluid equations for two species-particle, momentum and energy conservation equations.

Waves in Vlasov Plasma: Longitudinal and transverse waves in an unmagnetized plasma. Solution of initial value problem by Landau's method. Landau damping. Waves along and perpendicular to external magnetic field.

Fokker-Planck's equation. Electrical conductivity. Diffusion across a magnetic field. Velocity space diffusion.

Nonlinear wave propagation in a plasma. Solitons. KdV equations, Nonlinear Schrödinger equation.

References :

1. A.K. Nicholas and Alvin W. Trievelpiece, Principles of Plasma Physics, McGraw Hill Kogakusha, Ltd., Tokyo, New Delhi etc. (1973).
2. R.C. Davidson, Methods in Nonlinear Plasma Theory, Academic Press, New York and London.

18. MAGNETO HYDRO MECHANICS : 50 marks (4 CP) (Duration : 6 Months)

(Prerequisite : FLUID Dynamics)

1. Fundamental Equations. : Maxwell's equations.Moving deformable conductor.Basic equations of inviscid and viscous magnetohydrodynamics.Basic properties of the magnetic field.Alfven waves.

2. Boundary Conditions.

3. Incompressible magnetohydrodynamic flow.: Parallel steady flow.One dimensional steady viscous flow.Hartman flow.Couette flow.

4. Waves: One dimensional wave propagation.Simple waves.Shock waves.Hugoniot conditions.

5. Ordinary gas dynamics in steady flow.

References :

1. Magnetohydrodynamics and plasma dynamics, S I Pai.

2. Magnetohydrodynamics, Jeffreys.

19. Quantum Mechanics : 50 (4 CP) (Duration : 6 Months)

Need for a Quantum Theory, Bra-Ket vectors, Linear operators and their representations, Schroedinger theory, Uncertainty relations. Solutions of the Time-independent Schroedinger equation for potential walls and barriers. Transition from Classical to Quantum Mechanics. WKB approximations.

The Simple Harmonic Oscillator(SHO), Schroedinger equation in higher dimension. Operator methods, ladder operators - application to angular momenta and SHO. Hydrogen atom. Spin, Pauli matrices.

The time evolution operator, Schroedinger, Heisenberg and interaction representations.

Scattering experiments, Scattering cross section and Scattering amplitude, Potential scattering, Partial waves, Born approximation.

Relativistic wave equations - Klein- Gordon equation and probability density, Dirac equation, non relativistic limits. Lorentz covariance of the Dirac equation, Parity, Time-reversal and Charge-conjugation in Dirac equation, negative energy states, Hole theory.

References :

1. D.J. Griffiths, Introduction to Quantum Mechanics.
2. P.A.M. Dirac, The Principles of Quantum Mechanics.
3. W. Greiner and B. Müller, Quantum Mechanics.
4. L.I. Schiff, Quantum Mechanics.
5. B.H. Bransden and C.J. Joachain, Introduction to Quantum Mechanics.
6. E. Merzbacher, Quantum Mechanics.
7. R. Shankar, Principles of Quantum Mechanics.
8. L.E. Ballentine, Quantum Mechanics.
9. J.J. Sakurai, Advanced Quantum Mechanics.
10. J.J. Sakurai, Modern Quantum Mechanics.
11. W. Greiner, Relativistic Quantum Mechanics.

20. Mathematics of Finance & Insurance : 50 (4 CP) (Duration : 6 Months)

1. Corporate Finance,
2. Risk and Insurance,
3. Financial Econometrics,
4. Financial Derivatives, Mathematical Review
5. Discrete Time Finance, Continuous Time Finance
6. Risk Management
7. Computations in Finance
8. Optimisation Methods for Finance.

References:

1. Sheldon M. Ross, An Introduction to Mathematical Finance, Cambridge University.
2. Robert J. Elliott and P. Ekkehard Kopp, Mathematics of Financial Markats, Springer-Verlag, New York Inc.

21. **Fundamentals of Mathematical Biology : 50 (4 CP) (Duration : 6 Months)**

(To be offered only with NDSDE as major or minor elective)

1. Single species population dynamics.Linear and nonlinear first order discrete models.Differential equation models. Metapopulations.Delay effects.Structured populations.Euler-Lotka equations in discrete and continuous times.

2. Population Dynamics of interacting species.The Lotka Volterra equations.Modelling the predator functional response.Competition.Interacting metapopulations.

3. Infectious diseases.Simple epidemic and SIS diseases.SIR models.

4. Biological motion.Reaction Diffusion equations and traveling wave solutions.Spatial sread of epidemics.

5. Pattern formation.Turing instability and bifurcations.Activator inhibitor systems.

6. Tumour modeling.

References :

1. Essential Mathematical Biology, Brittn.

2. Mathematical Biology, (Vol I and II) Murrey.

3. Mathematics in population biology, Thieme.

Course Outcomes: The major electives are broadly classified into two groups or branches of Mathematics Pure or Applied. On successful completion of these courses in any one of the branches, students will be able to

1. Pursue advanced studies in these, take up research in any of these or in a related subject.
2. Apply these concepts to any pedagogical procedure.

<u>Course: 215 243</u>

Minor Elective Course II

1. Algebraic Topology & Category Theory : 50 Marks (4 CP) (Duration : 6 Months)

Homotopy Theory: (Marks - 20)

- **Homotopy and paths**
  Homotopy and Homotopy classes.Homotopy equivalences, Null homotopy, Relative homotopy, etc.. Composite of homotopic spaces.
  Contractible spaces, deformation, strong deformation retraction, etc.. Path-connected spaces - their union, intersection and continuous images.
  Product and inverse of paths. Homotopy of paths and products of homotopic paths.

- **Covering spaces and covering maps**
  Covering spaces and covering maps. Properties of covering maps.
  Path lifting property and Homotopy lifting theorem.

- **Fundamental group**
  Definition and verification. Homomorphism and isomorphism of fundamental groups.
  Fundamental groups of Circle.
  Fundamental groups of some known surfaces, e.g. Cylinder, punctured plane, Torus, etc.

Homology Theory: (Marks -20)

- **Finite Simplicial Complexes**
  Simplicial complexes.Polyhedra and Triangulation.
  Simplicial approximation, barycentric subdivision and simplicial approximation theorem.

- **Simplicial Homology**
  Orientation of simplicial complexes. Simplicial chain complexes, boundaries and cycles, homology groups – some examples.
  Induced homomorphisms. Reduced homology groups.
  Some applications, e.g., Invariance of dimension, no-retraction theorem, Brower’s fixed point theorem, etc.

Category theory : (Marks -10)

- **Category**

  Definition and Examples.Objects – initial, terminal and null objects.
  Morphisms – epi, monic, isomorphism, section and retraction, uniqueness of identity morphism.

- **Functor**

  Covariant and contravariant functors - Definition and Examples. Faithful and Full functors.Equivalent Categories
  Subcategory and Full subcategory, Dual category and principle of duality
  Natural Transformation – concept only with examples
  Fundamental groups as a covariant functor between the category of pointed spaces and base-point preserving continuous maps and the category of groups and homomorphisms
  Homology groups as a covariant functor between the category of all topological spaces and continuous maps and the category of groups and homomorphisms

References :

1. Algebraic Topology, A. Hatcher, Cambridge University Press
2. Algebraic Topology, W. Massey, Springer (GTM)
3. Algebraic Topology : A first course, M.J. Greenberg and J. R. Harper, Perseus books, Cambridge
4. Algebraic Topology : A Primer, S. Deo, Hindustan Book Agency (trim 27)
5. Categories for the Working Mathematicians (second edition), S. MacLane, Springer (GTM)

2. Fluid Dynamics - 50 Marks (4 CP) (Duration : 6 Months)

Introduction. Fluid Properties. Ideal Fluids. Viscous compressible and incompressible fluids. Non-Newtonian fluids.

Theory of Stress and Rate of strain for fluids. Isotropic fluids. Stokes' hypothesis for Newtonian fluids. Navier-Stokes'equation.

One dimensional inviscid incompressible flow.Euler's equation. Bernoulli equation.

Two dimensional and three dimensional inviscid incompressible flow.Basic Equations. Eulerian equations of motion. Circulation. Stokes'Theorem., Kelvin's theorem. Velocity potential. Irrotational flow. Integration of equations of motion.- Bernoulli's equation.Steady Motion.Stream function. Source and sink. Radial flow. Vortex flow.Doublet.Motion of solid bodies in Fluid.

Laminar flow of Viscous incompressible fluid. Similarity of Flows. Reynold's number. Flow between parallel plates. Couette flow. Plane Poiseuille flow. Steady flow in pipes.

Boundary layer concept. Boundary layers in two dimensional flow. Boundary layer along a flat plate.The Blasius solution.

Inviscid compressible flow. Field equations. Circulation. Propagation of small disturbance. Sound waves. Steady isentropic motion. Mach number and cone ; Bernolli's equation. Irrotational motion. Velocity potential . Bernoulli's equation for unsteady flow. Steady channel flow. Mass flux through a converging channel. Flow through nozzle.Normal shock waves.

Surface waves. Basic equations. Boundary condition. Progressive waves. Group velocity. Standing waves.

References :

1. L.M. Milne-Thomson, Theoretical Hydrodynamics.
2. P.K. Kundu and Iva M.Cohen, Fluid Dynamics, Har Court, India.
3. H. Lamb, Hydrodynamics, Dover Publication.
4. F. Chorlton, Text Book of Fluid Dynamics, CBS Publ.
5. H. Schlichting, Boundary Layer Theory, McGraw Hill.

Course Outcomes:

1. The minor electives provide a basic introduction to the specific branch of specializations (Pure/Applied) taken by the students.
2. A study of the minor electives enables the students to acquire mathematical tools/knowledge deemed essential for specializing in one branch.